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INTERACTIONS OF CORPORATE FINANCING AND INVESTMENT DECISIONS—IMPLICATIONS FOR CAPITAL BUDGETING

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I. INTRODUCTION

EVERYONE seems to agree that there are significant interactions between corporate financing and investment decisions. The most important argument to the contrary—embodied in Modigliani and Miller's (MM's) famous Proposition I—specifically assumes the absence of corporate income taxes; but their argument implies an interaction when such taxes are recognized. Interactions may also stem from transaction costs or other market imperfections.

The purpose of this paper is to present a general approach for analysis of the interactions of corporate financing and investment decisions, and to derive the approach's implications for capital investment decisions. Perhaps the most interesting implication is that capital budgeting rules based on the weighted average cost of capital formulas proposed by MM and other authors are not generally correct. Although the rules are reasonably robust, a more general "Adjusted Present Value" rule should, in principle, be used to evaluate investment opportunities.

The paper is organized as follows. Section II presents the framework for my analysis, which is a mathematical programming formulation of the problem of financial management. The conditions for the optimum and the implications for corporate investment decisions are derived. In Section III, the usual weighted average cost of capital rules are derived as special cases of the more general analysis. Section IV examines the errors that can occur if weighted average cost of capital rules are used in practice, and evaluates the rules' robustness. Finally, I discuss the Adjusted Present Value rule as an alternative for practical applications.

It must be emphasized that this paper is not intended to catalogue or deal with all possible interactions of financing and investment decisions; in other words, there is no attempt to specify the problem of financial management in

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full detail. I present an *approach* to analyzing interactions and a specific analysis of the most important ones.

Another limitation is that the model developed in the paper is static—that is, it does not consider how future financial decisions might respond to information which will become available in future periods. Instead the model specifies a financial plan which is optimal given that current expectations are realized. Since there is no assurance that sequential application of a static model constitutes optimal strategy under uncertainty, this paper is only of intermediate generality. In this respect it is no better or worse than the existing theory of financial management, which is likewise static.

The analysis is nevertheless of immediate interest. As far as I know the literature of finance contains no full analysis of the use of the weighted average cost of capital as a standard for capital budgeting. Most authors present sufficient conditions for its use, and are careful to warn the reader against assuming it to be generally valid.[1] They clearly regard it as a special case of some more general standard. But they do not specify the general standard in operational form, and therefore cannot offer much perspective on how special the special case really is, or on how dangerous it is to use the special rule generally. This paper, on the other hand, formulates a general model, states its implications in reasonably operational form (as the Adjusted Present Value rule), and then goes on to evaluate traditional procedures as special cases.

I do not mean this to minimize previous work on capital budgeting and the weighted average cost of capital, but simply to designate this paper's contribution.[2] My debt to the literature—particularly the Modigliani-Miller papers[3] —will be evident throughout.

II. Basic Framework

We will consider the firm's problem in the following terms. It begins with a certain initial package of assets and liabilities. For a brand-new firm, this will be simply money in the bank and stock outstanding. For a going concern, the package will be much more complicated. Any firm, however, has the opportunity to change the characteristics of its initial package by transactions in real or financial assets—i.e., by investment or financing decisions. The problem is to determine which set of current and planned future transactions will maximize the current market value of the firm. Market value is taken to be

1. See, for example, Miller and Modigliani [14], esp. pp. 346-343; Fama and Miller [7], pp. 170-175; Haley and Schall [9], ch. 13; Vickers [28] and Beranek [2].

Other authors who have expressed concern about the general applicability of the weighted average cost of capital include Robichek and McDonald [21], Arditti [1], and Tuttle and Litzenberger [26].

2. The paper most similar to this one is Beranek's [2]. He analyzes the necessary conditions for use of the weighted average cost of capital, and obtains a list of conditions essentially equivalent to the one presented below. However, his method of analysis is different, he is concerned only with the "textbook" formula (defined below), and he does not go on to evaluate robustness or propose general procedures. His paper does cover certain other issues not addressed here, for example the proper definition of "cash flow" for an investment project.

3. Miller and Modigliani [14] contains the most precise and compact exposition of their theory. See also their other papers, [15] and [16].

an adequate proxy for the firm's more basic objective, maximization of current shareholders' wealth.

This type of problem can be approached by (1) specifying the firm's objective as a function of investment and financing decisions and (2) capturing interactions of the financing and investment opportunities by a series of constraints.

General Formulation

Consider a firm which has identified a series of investment opportunities. It must decide which of these "projects" to undertake.[4] At the same time it wishes to arrive at a financing plan for the period $t = 0, 1, \ldots, T$. The financing plan is to specify for each period the planned stock of debt outstanding, cash dividends paid, and the net proceeds from issue of new shares.

Let: x_j = proportion of project j accepted.
y_t = stock of debt outstanding in t.
D_t = total cash dividends paid in t.
E_t = net proceeds from equity issued in t.
C_t = expected net after-tax cash inflow to the firm in t, with net outflow (i.e. investment) represented by $C_t < 0$.
Z_t = debt capacity in t, defined as the limit on y_t. Z_t depends on firm's investment decision,[5] i.e., $\delta Z_t/\delta x_j$ will normally be positive.

Also, let ψ equal ΔV, the change in the current market value of the firm, evaluated cum dividend at the start of period $t = 0$. In general, ψ is a function of the x's, y's, D's and E's.

The problem is to maximize ψ, subject to:

$$\phi_j = x_j - 1 \leqslant 0, \qquad j = 1, 2, \ldots, J. \tag{1a}$$

$$\phi_t^F = y_t - Z_t \leqslant 0, \qquad t = 0, 1, \ldots T. \tag{1b}$$

$$\phi_t^C = -C_t - [y_t - y_{t-1}(1 + (1 - \tau)r)] + D_t - E_t = 0 \quad t = 0, 1, \ldots, T. \tag{1c}$$

$$x_j, y_t, D_t, E_t \geqslant 0. \tag{1d}$$

The borrowing rate, r, is assumed constant for simplicity, as is the corporate tax rate τ. In general, r will be a function of the other variables.

This formulation of the firm's financial planning problem is perfectly general in the sense of not imposing restrictions (e.g., linearity) on the functions determining ψ or Z_t. It is by no means a *detailed* formulation. The maturity structure of the planned stock of debt is not treated, for example. Stock repurchases are not allowed. These "details," while important to the firm's overall financial plan,[6] are not critical to this paper.

4. Some projects may be future investment opportunities anticipated for $t = 1, 2, \ldots$. Accepting such a project does not imply immediate investment, but simply that the project is included in the firm's financial plan.

5. The limit may be imposed by capital markets or it may simply reflect management's judgment as to the best level of debt.

6. These "details" are considered in Myers and Pogue [20], who develop mathematical programming models for overall financial planning.

Conditions for the Optimum

Eqs. (1) define the nature of the interactions between the firm's financing and investment decisions. The effects of the interactions can be better understood by examining the necessary conditions for the optimal solution.

In order to simplify notation define $A_j \equiv \delta\psi/\delta x_j$, $F_t \equiv \delta\psi/\delta y_t$, $Z_{jt} \equiv \delta Z_t/\delta x_j$, and $C_{jt} \equiv \delta C_t/\delta x_j$. Also, note that each of the following equals 1: $\delta\phi_j/\delta x_j$, $\delta\phi_t^F/\delta y_t$ and $-\delta\phi_t^C/\delta y_t$. Finally, note that $\delta\phi_t^C/\delta x_j = -C_{jt}$. The shadow prices are λ_j for ϕ_j, λ_t^F for ϕ_t^F and λ_t^C for ϕ_t^C.

With these simplifications, the necessary conditions for the optimum can be written as follows. For each project:

$$A_j + \sum_{t=0}^{T} [\lambda_t^F Z_{jt} + \lambda_t^C C_{jt}] - \lambda \leqslant 0. \tag{2a}$$

For debt in each period,

$$F_t - \lambda_t^F + \lambda_t^C - [1 + (1 - \tau)r]\lambda_{t+1}^C \leqslant 0. \tag{2b}$$

For dividends in each period,

$$\delta\psi/\delta D_t - \lambda_t^C \leqslant 0. \tag{2c}$$

For equity issued in each period,

$$\delta\psi/\delta E_t + \lambda_t^C \leqslant 0. \tag{2d}$$

In each of these equations a strict equality holds if the corresponding decision variable is positive in the optimal solution.

Eq. (2a) is particularly interesting because it states the condition for evaluating a marginal investment in a project. Marginal investment is justified if project j's "Adjusted Present Value" (APV_j) be positive, i.e.,

Expand Investment if: $$APV_j \equiv A_j + \sum_{t=0}^{T} [\lambda_t^F Z_{jt} + \lambda_t^C C_{jt}] > 0. \tag{3}$$

In the optimal solution $APV_j = \lambda_j$ if the project is accepted ($x_j = 1$). If it is rejected ($x_j = 0$) then APV_j is negative and $\lambda_j = 0$. If it is partially accepted, then $APV_j = \lambda_j = 0$.

The term adjusted present value is used because in the optimal solution A_j, the project's direct contribution to the objective, is "adjusted for" the project's side effects on other investment and financing options. The side effects occur because of the project's effects on the debt capacity and sources/uses constraints.

Effects of financial leverage when dividend policy is irrelevant.—Suppose that dividend policy is irrelevant, in the sense that $\delta\psi/\delta E_t = \delta\psi/\delta D_t = 0$ for all T.[7] Then $\lambda_t^C = 0$, from Eqs. (1c) and (1d).

7. This defines irrelevance of dividend policy in the same way as Miller and Modigliani [14]. That is, *given* values for the x_j's and y_t's, a marginal change in D_t and an offsetting change in E_t will not affect shareholder's wealth.

Also, assume that $\delta\psi/\delta y_t$ is positive—which is realistic, given the tax deductibility of debt, regardless of whether one agrees with MM. Then the constraints ϕ_t^F will always be binding, Eqs. (2b) will be strict equalities, and λ_t^F for all t.

Substituting in Eq. (3),

$$APV_j = A_j + \sum_{t=0}^{T} Z_{jt}F_t. \tag{4}$$

Eq. (4) implies that APV_j, the contribution of a marginal investment in j to the firm's value, is measured by A_j, plus the present value of the additional debt the project supports.

Effects of dividend policy.—In practice, however, dividend policy may not be completely irrelevant. At very least, $\delta\psi/\delta E_t$ will be negative because of transaction costs associated with stock issues. It is not clear whether $\delta\psi/\delta D_t$ is positive, negative or zero in real life.[8]

Suppose that the optimal solution calls for an equity issue in a period t. Then $\lambda_t^C = -\delta\psi/\delta E_t$ and $\lambda_t^C > 0$. Examination of Eq. (2a) shows that this is reflected in the optimal solution in two ways. First, project j is penalized if $C_{jt} < 0$. On the other hand, the project is relatively more attractive if $C_{jt} > 0$: in this case the project generates funds and this reduces the need for a stock issue. Second, if the project contributes to debt capacity in t, this in turn reduces the need for the stock issue. This is evident in Eq. (2b), which shows that λ_t^F, the marginal value of debt capacity in t, depends on λ_t^C as well as on $\delta\psi/\delta y_t$.

The same type of interactions exists if dividends are paid in t and $\delta\psi/\delta D_t \neq 0$.

Conditions for independence of financing and investment decisions.—In a world with no taxes and perfect capital markets, both debt policy and dividend policy are irrelevant, i.e., $F_t = \delta\psi/\delta D_t = \delta\psi/\delta E_t = 0$. In this case the investment and financing decisions are independent, and APV_j equals simply A_j.

The independence of financing and investment decisions in a "pure MM world" is well known, but worth mentioning here because it reveals the economic interpretation of A_j. A_j is the contribution to firm value of marginal investment in project j, assuming all-equity financing and irrelevance of dividend policy. In a pure MM world that is all the financial manager needs to know. In effect, the APV concept first evaluates the project in this base case and then makes appropriate adjustments (via the shadow prices λ_t^F and λ_t^C) when debt and/or dividend policy is relevant and influenced by adoption of the project.

8. It can be argued that dividends decrease shareholder wealth because dividends are taxed more heavily than capital gains. On the other hand, it is possible that some investors positively prefer dividends because of the convenience of having a regular, "automatic" cash income, or for other reasons.

Although the matter of dividend policy is still controversial, recent evidence does not indicate that it is all that important, apart from the "informational content" of dividends which is not germane here. (See [15], pp. 367-70, [3] and [8] for empirical evidence consistent with the irrelevance of dividend policy.) Thus most of the analysis later in the paper assumes $\delta\psi/\delta D_t = 0$.

Decentralized capital budgeting systems.—The shadow prices λ_t^F and λ_t^C can, in principle, be used as a basis for a decentralized capital budgeting system. Consider the accept-reject decision on an individual project j. The first step is to estimate the project's contribution to firm value in the base case just described; call this ΔV_o. Then, given estimates of the project's year-by-year contribution to debt capacity (ΔZ_t) and after-tax cash flow (ΔC_t), the decision is:

$$\text{Accept if:} \qquad \Delta\psi = \Delta V_o + \sum_{t=0}^{T} (\Delta Z_t \lambda_t^F + \Delta C_t \lambda_t^C) > 0.$$

This may be written in the same form as Eq. (3), i.e.,

$$\text{Accept if:} \qquad APV_j = A_j + \sum_{t=0}^{T} (Z_{jt}\lambda_t^F + C_{jt}\lambda_t^C) > 0, \tag{3a}$$

with the understanding that APV_j, A_j, Z_{jt} and C_{jt} are interpreted as discrete amounts rather than partial derivatives.

The distinction between Eqs. (3) and (3a) is important. The discrete form (3a) is relevant for the simple accept-reject choice, given the project's scale. The continuous version (3) is relevant to the choice of optimal scale. The APV's computed according to the two formulas will not be the same unless the various partial derivatives in (3) are constants.

The remainder of the paper is concerned with the discrete accept-reject decision. This was done solely to simplify exposition. The reader can verify that the formal argument could just as well have been based on Eq. (3) as (3a); and that the major results also apply to the problem of determining optimal project scale.

It is, of course, necessary to take λ_t^F and λ_t^C as given regardless of the interpretation. This may be justified in two ways. One assumption is that project j is "small." Another is that the project, regardless of size, does not affect F_t, $\delta\psi/\delta D_t$ or $\delta\psi/\delta E_t$.

The second assumption requires further explanation. Inspection of Eqs. (2c) and (2d) shows that λ_t^C will equal either $\delta\psi/\delta D_t$ or $-\delta\psi/\delta E_t$ at the optimum, depending on whether the optimal plan calls for issuing stock or paying dividends. Thus, so long as $\delta\psi/\delta D_t$ or $\delta\psi/\delta E_t$ is constant, λ_t^C is independent of the decision to accept or reject j. This could be entirely realistic: $\delta\psi/\delta E_t$ might reflect a constant transaction cost per dollar of equity issued, for example.

If the debt constraint (1b) is binding, then Eq. (2b) will be an equality, and λ_t^F will be a constant if F_t, λ_t^C and λ_{t+1}^C are constants. Again, this seems plausible: for example, in an MM world with corporate taxes, F_t is simply present value of the tax shield generated per dollar of debt outstanding at t.

The practical implications of APV for project by project capital budgeting decisions are discussed in more detail later in the paper. Before that I will use the APV concept to analyze capital budgeting rules based on the weighted average cost of capital.

III. A Reexamination of the Weighted Average Cost of Capital Concept

Introduction and Definitions

It is widely accepted that the accept/reject decision for investment projects ought to be evaluated on a "DCF," or discounted cash flow, basis. This is done by one of two decentralized rules. The first is to compute project j's internal rate of return, R_j, from the formula

$$\sum_{t=0}^{T} \frac{C_{tj}}{(1+R_j)^t} = 0, \tag{5}$$

and to accept the project if R_j exceeds ρ_j^*, the "cost of capital" for j. The second rule is to compute the net present value of j's cash flows, discounted at ρ_j^*, and accept j if this figure is positive. Thus, j is accepted if

$$NPV_j = \sum_{t=0}^{T} \frac{C_{jt}}{(1+\rho_j^*)^t} > 0. \tag{6}$$

In either rule, ρ_j^* is the "hurdle rate" or minimum acceptable expected rate of return.

Comparing Eq. (3a) to (5) and (6), it is evident that NPV_j and APV_j are intended to measure the same thing: the net contribution of j to market value, taking account of the interactions of j with other investment and financing opportunities. There is always some value of ρ_j^* which will insure that $NPV_j = APV_j$, or that

$$\sum_{t=0}^{T} \frac{C_{jt}}{(1+\rho_j^*)^t} = A_j + \sum_{t=0}^{T} [\lambda_j^F Z_{jt} + \lambda_t^C C_{jt}] \equiv APV_j. \tag{7}$$

Eq. (7) may be regarded as an implicit definition of ρ^*. An analogous, but narrower definition is

$$\sum_{t=0}^{T} \frac{C_{jt}}{(1+\rho_j^*)^t} > 0 \quad \text{if and only if} \quad APV_j > 0. \tag{7a}$$

This interprets ρ_j^* simply as a hurdle rate, or minimum acceptable expected rate of return. A ρ_j^* derived from Eq. (7a) does not necessarily give a correct valuation (i.e., $NPV_j = APV_j$) for projects of more than minimum profitability. The conditions under which Eqs. (7) and (7a) are consistent are given later in this paper. For the moment we will work with Eq. (7a). The problem is, how should ρ_j^* be computed, if not directly from Eq. (7a)?

Of the many procedures for calculating ρ_j^*, two are of particular interest. The first is MM's. They propose[9]

$$\hat{\rho}_j^* = \rho_{oj}(1 - \tau L), \tag{8}$$

9. [15], p. 342. In MM's notation ρ^* is C(L) and ρ_{oj} is simply ρ_j.

where: ρ_{oj} = The appropriate discount rate assuming all-equity financing;

τ = The corporate tax rate;

L = The firm's "long-run" or "target" debt ratio; and

$\hat{\rho}^*_j$ = A proposed value for ρ^*_j.

MM interpret ρ_{oj} as the rate at which investors would capitalize the firm's expected average after-tax income from currently-held assets, if the firm were all-equity financed.[10] This would restrict application of the formula to projects whose acceptance will not change the firm's risk characteristics. (However, we will see that this is an unnecessarily narrow interpretation of the MM formula.)

The second proposed formula is:

$$\hat{\rho}^*_j = (1-\tau)r\frac{B}{V} + k\frac{S}{V} \tag{9}$$

where: r = the firm's borrowing rate at t = 0;

k = "the cost of equity capital"—that is, the expected rate of return required by investors who purchase the firm's stock;

B = market value of currently outstanding debt;

S = market value of currently outstanding stock, and

V = B + S, the total current market value of the firm.

I will refer to Eq. (9) as the "textbook formula," for lack of a better name. (The formula, or some variation on the same theme, appears in nearly all finance texts.)[11] It is not necessarily inconsistent with the MM formula, but it is recommended by many who explicitly disagree with MM's view of the world.

The task now is to determine what assumptions are necessary to derive Eqs. (8) and/or (9) from Eq. (3a), the general condition for the optimal investment decision. I will present a set of sufficient conditions, and then argue that, in most cases, the conditions are necessary as well.

Derivation of the MM Cost of Capital Rule

If MM's view of the world is correct, then the value of the firm will be V_0, the value of the firm assuming all-equity financing, plus PVTS, the present value of tax savings due to debt financing actually employed. Dividend policy is irrelevant. Assuming this view is correct, the objective function in the mathematical programming formulation is:

$$\psi = \Delta V_0 + \sum_{t=0}^{T} y_t F_t = \Delta V_0 + \sum_{t=0}^{T} \frac{y_t r\tau}{(1+r)^{t+1}}. \tag{10a}$$

10. Ibid., pp. 337, 340.

11. See Johnson [11], Ch. 11; Weston and Brigham [29], Ch. 11; Van Horne [27], Ch. 4.

That is, F_t is $r\tau$, the tax saving per dollar of debt outstanding in t, discounted to the present. (It is assumed that the interest is paid at $t + 1$.) Second, assume that

$$A_j = C_j/\rho_{oj} - I_j. \tag{10b}$$

That is, project j is expected to generate a constant, perpetual stream of cash returns.[12]

The third assumption is that undertaking project j does not change the risk characteristics of the firm's assets. That is,

$$\rho_{oj} = \rho_o, \tag{10c}$$

where ρ_o is the firm's cost of capital given all-equity financing.

Fourth, assume that project j is expected to make a permanent and constant contribution to the firm's debt capacity:

$$Z_{jt} = Z_j, \qquad t = 0, 1, \ldots, \infty. \tag{10d}$$

Finally assume

$$Z_j = LI_j, \tag{10e}$$

where L is the long-run "target" debt ratio which applies to the firm overall. Eq. (10e) implies that adoption of project j will not change this target.

Rewriting Eq. (3a) using Eqs. (10a) through (10e), we have:

$$\begin{aligned} APV_j &= \frac{C_j}{\rho_o} - I_j + LI_j \sum_{t=0}^{\infty} F_t \\ &= \frac{C_j}{\rho_o} - I_j + LI_j\tau. \end{aligned} \tag{11}$$

From Eq. (7a), the cost of capital is the project's internal rate of return (C_j/I_j) when $APV_j = 0$. Eq. (11) implies that this is given by MM's formula:

$$\rho_j^* = C_j/I_j = \rho_o(1 - \tau L).$$

Extension of MM's Result to Projects of Varying Risk

Let us make one further assumption, that ΔV_o is a linear function of the present values of accepted projects:

$$\Delta V_o = \sum_{j=1}^{J} x_j A_j. \tag{10f}$$

Eq. (10f) assumes that projects are *risk-independent*, in the sense that there are no statistical relationships among projects' returns such that some

12. If $C_{jt} = C_j$, a constant for $t = 1, 2, \ldots, \infty$, then Eq. (10b) simply states the project's net present value when discounted at ρ_{oj}, the "appropriate rate" for j given all-equity financing. However, MM interpret C_j as the expectation of the *mean* of the series $\tilde{C}_{j1}, \tilde{C}_{j2}, \ldots, \tilde{C}_{j\infty}$. See [14], p. 337. This does not require that C_{jt} is constant, but there must be conditions to insure that this mean is finite. The reader may choose the interpretation he likes best. The form of the argument to follow is not affected.

combinations of projects affect stock price by an amount different than the sum of their present values considered separately. In particular, risk-independence implies that there is no advantage to be gained by corporate diversification. Risk-independence is a necessary condition for equilibrium in perfect security markets.[13]

Eq. (10f) also assumes that projects are "physically independent" in the sense that there are no *causal* links between adoption of project j and the probability distribution of cash returns to other projects—that is, it rules out "competitive" or "complementary" projects. Such interactions make it impossible to specify an unique hurdle rate for project j, since the minimum acceptable rate of return on j may depend on whether or not other projects are accepted. However, I am not concerned with this problem in this paper.

Let us adopt Eq. (10f) and drop Eqs. (10c) and (10e). We can recalculate the minimum acceptable rate of return on the project.

$$\rho^*_j = \rho_{oj}(1 - \tau Z_j/I_j). \tag{12}$$

This has the same form as Eq. (8) but is not restricted to projects within a single risk class. However, it is not plausible to identify Z_j/I_j, project j's *marginal* contribution to debt capacity, with L, the firm's overall target capitalization ratio. Presumably Z_j/I_j will be more or less than L, depending on the risk or on other characteristics of the project in question.

In short, MM's formula can be extended to independent projects which differ in risk and in their impact on the firm's target debt ratio.

What If Investment Projects Are Not Perpetuities?

So far we have established that Eqs. (10a, b, d and f) are sufficient for the generalized MM formula, Eq. (12). Eqs. (10a) and (10f) are clearly necessary as well. But what about (10b) and (10d), which require all projects to be perpetuities?

In general, they are necessary: Eq. (13) does not give the correct "hurdle rate" for projects of limited life.[14] (The question of whether the resulting errors are serious is taken up in the next section.)

This can be shown by a simple example. Consider a point-input, point-output project requiring an investment of I_j and offering an expected cash flow of C_{j1} in $t = 1$, and $C_{jt} = 0$ for $t > 1$. Assume $\rho_{oj} = \rho_o$ and $Z_{j1} = LI_j$ (and, of course, $Z_{jt} = 0$ for $t > 1$). Then

$$APV_j = \frac{C_{j1}}{1 + \rho_o} - I_j + LI_j\left(\frac{r\tau}{1 + r}\right).$$

The internal rate of return on the project is given by $R_j = C_{ij}/I_j - 1$, and the cost of capital is given by R_j when $APV_j = 0$. Thus

13. Myers [19] and Schall [24]. See Merton and Subramanyam [13] for a recent review of work relating to this aspect of capital market equilibrium.

14. Of course, the importance of project life has been recognized by MM (in [17], for example) and others (e.g., [1], [2], and [7], esp. p. 173n). But the implications for the cost of capital ρ^*_j have not been developed in the literature.

$$\rho_j^* = \rho_o - L r \tau \left[\frac{1+\rho_o}{1+r}\right]. \tag{13}$$

Eqs. (12) and (13) are equivalent only in the uninteresting case of $\rho_o = r$.

The Textbook Formula

Let us reconsider Eq. (9),

$$\hat{\rho}_j^* = r(1-\tau)B/V + k(S/V). \tag{9}$$

Probably it is intuitively clear from the foregoing that $\hat{\rho}_j^* = \rho_j^*$ only under very restrictive assumptions. First, let us assume that Eqs. (10a) through (10e) hold.[15] (Remember that (10a) implies that dividend policy is irrelevant.) Also, assume that

$$V_o = \frac{C}{\rho_o}. \tag{14a}$$

That is V_o, the current market value of the firm if it were all equity financed, is found by capitalizing the firm's after-tax operating income at ρ_o. C is, of course, calculated assuming all-equity financing. Also Eq. (14a) presumes $C_t = C$, $t = 1, 2, \ldots, \infty$.[16]

Finally, assume that the firm is already at its target debt ratio.

$$L_j = B/V. \tag{14b}$$

Note that Eqs. (14a) and (14b) constrain the initial characteristics of the firm's assets and financing mix, whereas the assumptions underlying MM's cost of capital formula relate only to the marginal effects of adopting the project in question.

Now the task is to show that $\hat{\rho}_j^* = \rho_j^*$ under assumptions (10a-e) and (14a-b). Note first that the sum of payments to bondholders and earnings after interest and taxes is $rB + kS = C + \tau rB$, so that $C = r(1-\tau)B + kS$ and

$$V = \frac{C}{\hat{\rho}_j^*}. \tag{15}$$

In an MM world, V is also given by

$$V = \frac{C}{\rho_o} + \tau B, \tag{16}$$

which is equivalent to Eq. (11). We now combine Eqs. (15) and (16) and solve for $\hat{\rho}_j^*$:

15. Eq. (10f) is not relevant, since Eq. (10c) implies that project j will not change the risk characteristics of the firm's assets.

16. Alternatively, we could regard C as the expected value of the mean of the stream $\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_\infty$. See fn. 12 above.

$$\hat{\rho}^*_j = \rho_o\left(1 - \tau B\left(\frac{\hat{\rho}^*_j}{C}\right)\right).$$

But $\hat{\rho}^*_j/C = 1/V$, and $B/V = L_j$, so

$$\hat{\rho}^*_j = \rho_o(1 - \tau L_j),$$

which was previously demonstrated to be the correct value.

Thus we have shown that the textbook formula gives the correct cutoff rate for projects under a long list of assumptions, one of which is that MM are correct. However, it can be readily shown that the formula is correct even if MM are wrong, providing the other assumptions hold.[17]

To summarize, the textbook formula gives the correct hurdle rate if:

1. The project under consideration offers a constant, perpetual stream of cash flows, and is expected to make a permanent contribution to debt capacity.
2. The project does not change the risk characteristics of the firm's assets.
3. The firm is already at its target debt ratio, and adoption of the project will not lead the firm to change the ratio.
4. The firm's currently-held assets are expected to generate a constant after-tax cash flow C per annum. This stream is expected to continue indefinitely.

The last of these assumptions may be surprising. We know from Eq. (7) or (7a) that the cost of capital ρ^*_j does not depend on the pattern of expected cash flows offered by the firm's existing assets. But it can be readily shown that the pattern does affect the observed value $\hat{\rho}^*_j$. Let us assume that the life of the firm's existing assets will end at the close of $t = 1$. Retain all the other assumptions for the textbook formula, and assume MM are right. We must thus replace Eq. (14a) with

17. If MM are wrong, then

$$V = \frac{C}{\rho_o} + B\sum_{t=0}^{\infty} F_t \tag{N1}$$

where F_t reflects not only the present value of tax savings but also the impact of any relevant market imperfections. Then it is readily shown that the true cost of capital is

$$\rho^*_j = \rho_o\left(1 - L_j\sum_{t=1}^{\infty} F_t\right). \tag{N2}$$

Proceeding as before, we observe

$$V = \frac{C}{\rho_o} + B\sum_{t=0}^{\infty} F_t = \frac{C}{\hat{\rho}^*_j}.$$

Solving for $\hat{\rho}^*_j$ we find it to be the value given by Eq. (N2).

Incidentally, Eq. (N2) is an attractive alternative for those who disagree with MM but are also uncomfortable with the textbook formula.

$$V_o = \frac{C_1}{1+\rho_o}. \tag{17}$$

Also,

$$V = V_o + PVTS = \frac{C_1}{1+\rho_o} + \frac{\tau r L V}{1+r}. \tag{18}$$

Observe that rB + kS, the total return received by stock and bondholders is equal to $C_1 + \tau rB - V$. This implies

$$r(1-\tau)B + kS = C_1 - V = \hat{\rho}^*_j V.$$

Thus:

$$V = \frac{C_1}{1+\rho_o} + \frac{LV\tau r}{1+r} = \frac{C_1 - V}{\hat{\rho}^*_j}.$$

Now we can solve for $\hat{\rho}^*_j$:

$$\hat{\rho}^*_j = \rho_o - L\tau r\left(\frac{1+\rho_o}{1+r}\right). \tag{21}$$

This establishes that the pattern of expected cash flows offered by the firm's existing assets does affect the observed value $\hat{\rho}^*_j$, which in this case is simply the hurdle rate for a one-period project. (See Eq. (13).)[18]

Summary

Table 1 summarizes the necessary and sufficient conditions for the derivation of MM's cost of capital formula, the generalized MM formula, and the textbook formula. Obviously, these conditions are quite stringent, particularly in the case of the textbook formula. The next section considers whether serious errors result when the conditions do not hold.

IV. How Robust are the Weighted Average Cost of Capital Formulas?

Introduction

The derivation of a cost of capital ρ^*_j for practical use involves two steps. The first is to measure the ρ_{oj}'s, the market opportunity costs of investing in assets of different levels of risk. The second is to adjust these opportunity costs to reflect the tax effects of debt financing, transaction costs of external financing, etc. These two steps are explicit in the MM cost of capital formulas and implicit in the textbook formula.

The difficulties in step (1) are notorious. My experience suggests that the confidence limit on empirical and/or subjective estimates of ρ_{oj} is at least a

18. This leads to the conjecture that the textbook rule is valid if, instead of Eqs. (14a) and (14b), it can be assumed that the stream of expected cash flows is strictly proportional over time to the cash flows of the firm's existing assets. However, I have not proved this generally. In any case, if $C_{jt} \doteq h_j C_t$, where h_j is a constant, then we hardly need worry about the cost of capital. It suffices to determine whether $I_j < h_j V$, where I_j is the initial investment required for the project.

TABLE 1
NECESSARY AND SUFFICIENT CONDITIONS FOR COST OF CAPITAL FORMULAS

(Equation)	Condition	Formula MM	Generalized MM	Textbook
—	Dividend policy irrelevant	x	x	x
(11a)	Leverage irrelevant except for corporate income taxes	x	x	
(11b)	Investment projects are perpetuities	x	x	x
(11c)	Project does not change firm's risk characteristics	x		x
(11d)	Project makes a permanent contribution to debt capacity	x	x	x
(11e)	Acceptance of project does not lead to shift of target debt ratio	x		x
(11f)	Risk-independence	n.a.*	x	n.a.*
(15a)	Firm's assets expected to generate a constant and perpetual earnings stream			x
(15b)	Firm is already at target debt ratio			x

* n.a. = not applicable.

percentage point under the most favorable conditions. This implies a certain tolerance for minor errors in step (2). How serious can these errors be, considered relative to the possible errors in step (1)? The purpose of this section is to begin exploring this question.

There are eight distinct assumptions listed in Table 1. Any one or any combination of them could be violated in practice. It is not feasible to compute the error for all possible cases. Instead, I will focus on assumptions (10b) and (10d), which require that the project being considered is expected to make a permanent contribution to the firm's earnings and debt capacity. These are the only assumptions necessary for all three cost of capital rules.

The decision to concentrate on (10b) and (10d) was based on several points concerning the other assumptions.

1. Assumptions (10a) and (10f) were not considered because they may well hold in fact. The empirical evidence to date does not lead to rejection of the MM and risk-independence hypotheses, and a strong theoretical case can be made for them.[19]

19. Probably the most extensive and sophisticated test or the MM propositions is MM's own study of the electric industry [15]. This study supports their theory. There is controversy about MM's tests: see, for example, Robichek, McDonald and Higgins [22], Crockett and Friend [5], Brigham and Gordon [4] and Elton and Gruber [6]. There clearly is room for a good deal more work, but despite the problems, we can at least say that recent work is not inconsistent with the MM hypotheses.

The proposition of risk independence is even harder to test directly. There is circumstantial evidence indicating that diversification is not an appropriate goal for the firm—for example, if investors were willing to pay for diversification would not closed-end mutual funds sell at a premium over asset value? And there is certainly no lack of diversification opportunities—even the small investor can buy mutual funds.

2. Assumptions (10c) and (10e) are not necessary for the generalized MM formula. It is clear, of course, that substantial errors can result if either assumption is violated and either the original MM or textbook formula is used. But the extent of the error can be readily estimated by comparing the rate obtained from the original MM formula or textbook formula with the rate obtained from the MM formula.

   Note that if (10c) does not hold, (10e) is not likely to hold either. A low-risk project will probably also make a large contribution to the firm's debt capacity.
3. Assumptions (14a) and (14b) were not analyzed explicitly because the results of violating them will be similar in magnitude to the results of violating (10b) and (10d) respectively.

Effects of Expected Project Life

I will start with an extreme case, by comparing the cost of capital obtained via the MM rule with the true cost of capital for a one-period project.

Remember that the MM formula generates a proposed value $\hat{\rho}^*_j$, given by

$$\hat{\rho}^*_j = \rho_{oj}(1 - \tau L). \tag{8}$$

The correct value is

$$\hat{\rho}^*_j = \rho_{oj} - Lr\tau\left(\frac{1 + \rho_{oj}}{1 + r}\right). \tag{13}$$

For simplicity, we will omit the j's henceforth.

Comparing Eqs. (9) and (14), it is clear that $\rho^* > \hat{\rho}^*$ for reasonable values of L and ρ_o. The error, ϵ, is

$$\epsilon = \rho^* - \hat{\rho}^* = L\tau\left[\rho_o - r\left(\frac{1 + \rho_o}{1 + r}\right)\right]. \tag{22}$$

From this we see that $\delta\epsilon/\delta L > 0$ and that $\delta\epsilon/\delta\rho_o = L\tau(1 - \frac{r}{1 + r}) > 0$. The error in $\hat{\rho}^*$ is highest for high-risk projects that can be heavily debt financed.

Table 2 consists of values of ϵ computed for values of ρ_o from 8 per cent to 25 per cent and for debt to value ratios of 10 to 60 per cent. ϵ ranges from .1 per cent to about 5 per cent. The errors shown in the bottom right of the table are dramatic, but the figures in the center, top right and bottom left of the table reflect the most reasonable combinations of capitalization rates and debt ratios. These errors are on the order of one percentage point, which is not serious. (Note that a one percentage point error in ρ^* for a one period project implies an error in NPV of only about one per cent of project investment.)

Tests of the "capital asset pricing model" of Sharpe [25], Lintner [12] and Mossin [18] may shed light on the risk-independence hypothesis. (The capital asset pricing model is sufficient but not necessary for risk-independence.) The empirical work to date indicates that the capital asset pricing model is probably an oversimplification, but it is too early to say for sure. Jensen [10] reviews the theory and evidence.

TABLE 2

ERROR[a] IN MM COST OF CAPITAL FORMULA FOR ONE-PERIOD PROJECT

ρ_0, Cost of Capital for all-equity financing	L, Target Debt Ratio					
	.1	.2	.3	.4	.5	.6
.08	.000	.001	.001	.002	.002	.003
.10	.002	.003	.004	.006	.007	.008
.12	.002	.005	.007	.009	.012	.014
.16	.004	.008	.013	.017	.021	.025
.20	.006	.012	.018	.024	.031	.037
.24	.008	.016	.024	.032	.040	.048

[a] Rounded to third decimal place.
The risk-free rate is assumed to be $r = .07$ and the tax rate is assumed to be $\tau = .5$.

Evidently the error in ρ^* will be smaller, the longer the life of the project under consideration. However, a more important statistic is the error in NPV caused by use of an incorrect discount rate. This error at first increases as a percentage of project investment as project life is lengthened but finally decreases to zero for projects of infinite life.

Take, for example, a ten-year opportunity requiring investment of $1000 at $t = 0$ and offering a constant expected cash return for $t = 1, 2, \ldots, 10$. Table 3 shows the difference between (1) NPV computed using $\hat{\rho}^*$ from the MM or textbook formulas and (2) the projects' true APV.[20] The errors in this case are more serious than for a one-period project, but still on the order of two to four per cent of project investment.

These mental experiments indicate that the MM or textbook rules are reasonably robust with respect to variations in project life. An unqualified endorsement is not in order, however. First, use of these rules makes invest-

TABLE 3

ERROR[a] IN INDICATED NPV FROM USING MM COST OF CAPITAL FORMULA TO EVALUATE TEN-PERIOD PROJECT REQUIRING $1000 INVESTMENT[b,c]

ρ_0, Cost of Capital for all-equity financing	L, Target Debt Ratio					
	.1	.2	.3	.4	.5	.6
.08	$ 2	4	6	8	10	12
.10	5	9	15	20	26	32
.12	7	14	21	29	38	47
.16	9	19	30	42	54	68
.20	10	22	34	48	63	79
.24	11	23	36	50	67	84

[a] Project investment and cash flows were taken as given—see note b below. Figures shown are NPV computed at $\hat{\rho}^* = \rho_0(1 - T_cL)$, minus APV. Figures rounded to nearest dollar.

[b] The risk-free rate is assumed as $r = .07$ and the tax rate is assumed to be $\tau = .5$. The project's expected cash flows are $150.00 per period from $t = 1$ to $t = 10$, and zero for $t > 10$.

[c] APV calculated by procedure described at pp. 27-28 below.

20. APV was computed by the procedure described at pp. 27-28 below.

ment projects look more valuable than they actually are. Second, the seriousness of the error depends on the specific pattern of project cash flows; the fact that the error was minor for the cases investigated does not prove the financial manager is safe in using the rules for projects with unusual patterns of cash flow over time.

Weak and Strong Definitions of the Cost of Capital

Let us suppose we have used Eq. (7a) to calculate the correct value of ρ^* for project j using the weak definition of the cost of capital. Then we can be assured that the project is a good one if NPV computed at ρ^* is positive. However, if projects j and k are mutually exclusive, and both have positive NPV's computed at correct hurdle rates, it is *not* generally correct to accept j over k if $NPV_j > NPV_k$. The general rule is to compare APV_j and APV_k, but Eq. (7a) insures that $NPV_j = APV_j$ only when $APV_j = 0$.

Under what conditions will discounting at the correct hurdle rate give correct value for NPV_j when project j is more than minimally profitable? To put it another way, under what conditions is ρ^*, calculated according to the strong definition of Eq. (7), independent of project profitability?

Assume the pattern of project cash flows over time is fixed. That is, $C_t = \gamma_t C$, for $t \geqslant 1$, where the γ_t's are constants, and C is varied to reflect changes in project profitability. C_o, project investment, is a fixed number, and the project's "risk class" is taken as given.

Now consider Eq. (7):

$$\sum_{t=0}^{T} \frac{C_t}{(1+\rho^*)^t} = A_j + \sum_{t=0}^{T} [\lambda_t^F Z_{jt} + \lambda_t^C C_{jt}] = APV_j. \tag{7}$$

Now divide through by C, after representing A_j by the usual present value formula and subtracting C_o from both sides

$$\sum_{t=1}^{T} \frac{\gamma_t}{(1+\rho^*)^t} = \sum_{t=1}^{T} \frac{\gamma_t}{(1+\rho_o)^t} + \sum_{t=0}^{T} \left[\lambda_t^F \left(\frac{Z_{jt}}{C}\right)\right] + \lambda_o^C \left(\frac{C_o}{C}\right) + \sum_{t=1}^{T} \lambda_t^C \gamma_t. \tag{23}$$

Eq. (23) is an alternative definition of ρ^*. C can be eliminated if the following conditions hold.

1. The project's expected period-by-period contributions to debt capacity are proportional to C.
2. $\lambda_0^C = 0$. That is, the dividend reduction (or stock issue) required to supply equity financing for the project must not affect shareholders' wealth; $\delta\psi/\delta D_o$ (or $\delta\psi/\delta E_o$) must equal zero.
3. The shadow prices λ_t^F and λ_t^C must be independent of C.

The third condition is not implausible.[21] In any case, a violation of it will under-

21. See p. 6 above.

mine *any* decentralized capital budgeting rule. The second condition could be handled by redefining the project's required investment as $C_o(1 + \lambda_o^C)$, in which case the term $\lambda_o^C(C_o/C)$ would not appear in Eq. (23).

The first condition is more interesting. It will clearly *not* be satisfied if the firm's target debt ratio is specified in book terms, since in that case Z_{jt} is independent of C. It will be satisfied if planned debt is related to the project's *market value*. Specifically, suppose

$$Z_t = L_t(APV_t - C_t), \tag{24}$$

that is, the firm's planned borrowing in t is a given proportion L_t of APV_t, the project's contribution to firm value in t, after receipt of project cash flow in t.[22] In this case ρ^* is independent of C, as is shown in the Appendix.

Thus, we can add one more condition to the list of assumptions in Table 1: weighted average cost of capital formulas give correct project valuation (i.e., NPV @ $\hat{\rho}^*$ = APV) only if the firm's target debt levels are specified in market value terms, or if the project has APV = 0. If the firm specifies target debt levels in book terms, then discounting at the $\hat{\rho}^*$'s will, other things equal, overstate project APV if APV > 0, and understate APV if APV < 0.[23]

The magnitude of possible error may be illustrated by the following numerical example. A project requires investment of $C_o = -1000$, but offers a constant expected stream of cash returns, C. The target debt ratio is L = .4, $\rho_o = .12$ and $\tau = .5$. Dividend policy is irrelevant as is financial leverage except for corporate taxes.

In this case the correct ρ^* is given by Eq. (8), the MM formula. It is

$$\rho^* = \rho_o(1 - \tau L) = .12(1 - .5(.4)) = .096.$$

The project's APV is $-1000 + C/.096$.

Another alternative would be to compute APV directly, as the sum of the project's value assuming all equity financing and the present value of tax savings generated due to the project's contribution to debt capacity.

$$APV = \left(\frac{C}{\rho_o} + C_o\right) + \tau L(APV - C_o)$$

$$APV = \left(\frac{C}{.12} - 1000\right) + .5(.4)(APV + 1000).$$

22. An alternative rule,

$$Z_t = L_t(A_t - C_t)$$

would also allow ρ^* to be calculated independent of project profitability. A_t is defined by

$$A_t = \sum_{1-t}^{T} \frac{C_1}{(1 + \rho_o)^{1-t}}.$$

23. This, of course, assumes debt capacity is valuable. It should also be noted that the level of profitability consistent with APV = 0 will depend on whether debt targets are in book or market terms, since book and economic depreciation are not generally equivalent.

Now compare this result to APV if the target debt ratio is set in book terms:

$$\underset{\substack{\text{(book debt}\\ \text{target)}}}{\text{APV}} = \left(\frac{C}{.12} - 1000\right) + .5(.4)(1000).$$

The difference is .5(.4)(APV), that is 20 per cent of APV. This seems to me a serious error—although the error would be less for shorter-lived projects or for lower debt ratios.

Summary

Whether capital budgeting rules based on the weighted average cost of capital deserve the label "robust" depends entirely on one's tolerance for error. I would consider the generalized MM formula acceptably accurate for accept-reject decisions on run-of-the-mill projects. The original MM formula is acceptably accurate if attention is restricted to projects which do not shift the firm's risk class or target debt ratio. The textbook rule is inferior on all counts[24] if used directly[25] as a standard for investment decisionmaking.

Of course, it is always possible to find the correct value of ρ^* from Eqs. (7) or (7a). The procedure is relatively simple: first, calculate APV, and then find the discount rate which gives the correct NPV, i.e., NPV $=$ APV. But once a project's APV is known, there is no need to calculate its ρ^*; it is sufficient to know whether APV > 0. Why not forget about ρ^* and use APV as the capital budgeting standard? The next section considers whether this is a practical alternative.

V. Adjusted Present Value as an Operational Capital Budgeting Standard

An alternative procedure is clearly needed for cases in which one or more of the assumptions underlying the weighted average cost of capital formulas are seriously violated. The natural choice is to accept project j if its adjusted present value is positive, i.e., if:

24. It might be argued that the textbook formula should be used by those who disagree with MM. But it is entirely feasible to develop a formula exactly like the MM formulas except for the assumed benefit of debt financing. See Eq. (N2), fn. 17 above.

25. The textbook formula may be helpful in *measuring* ρ_0. Suppose a firm can estimate k, the expected rate of return investors in the firm's stock. Then ρ^* can be directly calculated.

$$\hat{\rho}^* = (1 - \tau)\, r\frac{B}{V} + k\frac{S}{V} \tag{9}$$

This value is not an appropriate standard for capital budgeting unless a variety of conditions hold, among them the equality of target and actual debt ratios ($L = B/V$). However, if the target were B/V, then, assuming MM are correct,

$$\hat{\rho}^* = \rho_0\left(1 - \tau\left(\frac{B}{V}\right)\right) = (1-\tau)r\frac{B}{V} + k\frac{S}{V} \tag{N3}$$

So an estimate of $\hat{\rho}^*$ can be translated into an estimate of ρ_0.

$$\text{APV}_j = A_j + \sum_{t=0}^{T} Z_{jt}F_t > 0. \tag{4}$$

In the event that dividend policy is relevant and/or there are significant transaction costs in new external financing, the criterion should be expanded to:

$$\text{APV}_j = A_j + \sum_{t=0}^{T} [Z_{jt}\lambda_t^F + C_{jt}\lambda_t^C] > 0. \tag{3a}$$

Calculating APV

The general procedure for calculating APV is obvious from the definition of the concept. First, A_j, the project's base case value, has to be calculated. This can be done by the usual NPV formula, except that the discount rate is ρ_{oj}. However, if the discounting procedure is inappropriate,[26] then any other procedure for estimating value may be followed. (This is a further advantage of the APV rule.)

The next step is to estimate the project's contribution to firm debt capacity, assign a value to this contribution and add it to A_j. (In an MM world, this amounts to adding the present value of tax shields generated by debt supported by the project. However, the APV rule does not *assume* MM are right.)

The third step is to determine whether the marginal source of equity financing is additional retained earnings, additional stock issue or a reduction in share repurchases. If there are special costs or benefits associated with the source (vs. the base case of irrelevance of dividend policy) then these can be incorporated in the λ_t^C's and the project value adjusted by adding ΣC_{jt}.

Perhaps the most difficult step in this process is to determine the Z_{jt}'s. This is simple if the firm's debt limits are determined by book debt ratios, since the Z_{jt}'s are then fixed ex ante and independent of project profitability or value (given book depreciation policy).

Calculating a project's adjusted present value turns out to be a moderately complex task when Z_{jt} is related to market value. The problem is that APV_{jo}, adjusted present value of project j as of $t = 0$, depends on estimated values of APV_{jt} for later periods. If the horizon is $t = T$, we have to calculate $\text{APV}_{j,T-1}$, $\text{APV}_{j,T-2}$, etc., and then finally APV_{jo}. For present purposes we will drop the j's and assume that Z_t is a constant proportion L of $\text{APV}_t - C_t$, except that $Z_T = 0$. We also assume that λ^C's are zero. That is, it is assumed that the firm plans to readjust its debt level at the end of every period in terms of its value at that time, and that this level is maintained during the next period. Also we assume that MM are right, i.e., that $F_t = \dfrac{\tau r}{(1+r)^{t+1}}$. Thus

26. Due to the problems cited by Robichek and Myers [23], for example.

$$APV_0 = A_0 + \sum_{t=0}^{T-1} \frac{\tau r L(APV_t - C_t)}{(1+r)^{t+1}}. \tag{24}$$

Let $f = \dfrac{\tau r L}{1+r}$. Then

$$\begin{aligned} APV_{T-1} &= A_{T-1} + f(APV_{T-1} - C_{T-1}) \\ &= \frac{A_{T-1} - fC_{T-1}}{1-f} = \frac{C_T/(1+\rho_0)}{1-f}. \end{aligned} \tag{25}$$

Having calculated APV_{T-1} we can determine APV_{T-2} from:

$$APV_{T-2} = A_{T-2} + f(APV_{T-2} - C_{T-2}) + \frac{f}{1+r}(APV_{T-1} - C_{T-1}). \tag{26}$$

The general formula for any interim period $t = T - S$ is

$$APV_{T-S} = A_{T-S} + f(APV_{T-S} - C_{T-S}) + f\left[\sum_{t=T-S+1}^{T-1} \frac{(APV_t - C_t)}{(1+r)^{t-T+S}}\right]. \tag{27}$$

Of course Eq. (26) reduces to (24) when $S = T$.

This backwards-iteration procedure is tedious to work through manually, but I did not find it difficult to construct a computer program to do the calculations. Also, note that the calculations are done as a by-product of the linear programming models of Myers and Pogue.[27]

Comments

This calculation procedure leads to two interesting theoretical observations. First, we might question the rationality of planning to keep L constant over time. Consider an equity investor in a single project firm with bonds B outstanding and equity worth S. Note $V = B + S = APV = A + PVTS$, where PVTS is the present value of the tax shield due to debt financing. The equity may be thought of as a portfolio long in assets A, long in the tax shield PVTS and short in debt B. If PVTS and B are equivalent-risk assets, then the portfolio weights are as follows.

Long position in firm's assets:

$$\frac{A}{APV}\left(1 + \frac{B}{S}\right)$$

Short position in B, net of PVTS:

$$\frac{B}{S} - \frac{APV - A}{APV}\left(1 + \frac{B}{S}\right)$$

27. See [20].

where $APV - A \equiv PVTS$. If the investor wants to maintain a constant degree of financial risk, he will set $\frac{A}{APV}(1+\frac{B}{S}) = Q$, a constant. This implies

$$L_t \equiv \frac{Z_t}{APV_t} = 1 - \frac{1}{Q}\left(\frac{A}{APV}\right). \tag{28}$$

Thus we would expect L_t to decline as the project ages and $\frac{A}{APV}$ approaches 1. The empirical prediction is that firms with long-lived assets would have higher debt ratios.

The second observation concerns the discount rate used in computing PVTS. I have followed MM who argue that it should be the risk-free rate r. This is clearly appropriate if the debt levels Z_{jt} are fixed at $t = 0$ and not changed thereafter. If Z_{jt} is determined by book debt ratios, for example, then there is no uncertainty about future tax shields, since there is no uncertainty about future profitability. On the other hand, suppose management wishes to maintain a constant ratio $L_j \equiv \frac{Z_{jt}}{APV_{jt}}$ over time. This means that Z_{jt} is a random variable that is perfectly correlated with APV_{jt} and *thus has the same risk characteristics*. The implication is that PVTS should be computed at ρ_o, not r. The intuitive meaning of this is that, although the tax shield associated with any debt instrument is safe, the aggregate value of instruments obtainable is uncertain. We have in effect a compound lottery; the fact that the second stage is risk-free does not mean that the lottery itself is safe.

There are a number of reasons why firms do not immediately adjust the value of bonds outstanding to every change in project or firm value. But to the extent that future debt capacity is contingent on future value of the firm's assets, the debt tax shield takes on the assets' risk characteristics. This is another reason why use of, say, the generalized MM formula for ρ^* would tend to overestimate APV. (So would use of Eq. (3a) or (4) unless the F_t's were computed using a discount rate greater than r.)

Using APV

Objections to the practical use of APV might be made on the basis of lack of realism, increased complication, the unfamiliarity of managers with the concept and the deficiencies of a static model.

Realism is not a valid objection relative to traditional rules. As was shown above, APV is a more general concept and therefore is more adaptable to whatever assumptions are considered "realistic."

The extra complication of the APV rule is a valid point for decisionmakers concerned with run-of-the-mill projects. However, for large and/or unusual projects the extra effort involved in using APV does not seem large relative to the magnitude of errors that might be avoided.

Lack of familiarity is a valid temporary objection. Understanding and interpreting the concept does require financial sophistication—although I have found it easier to explain to beginning finance students than to sophisticated financial managers who have "learned" the concept of discounting at "the" cost of capital.

The static assumptions underlying APV are a real liability, although it is no worse than traditional approaches on this dimension. Whether to advise use of APV in spite of its static assumptions is a question that requires balancing possible errors due to the deficiencies of APV against the improved decisions stemming from its use. But it would seem that anyone who now advises use of traditional capital budgeting rules should be willing to advise a definite improvement.

Perhaps the greatest advantage of the APV concept is that it guides the corporate financial manager through various problems that turn into a can of worms when analyzed by any approach relying on the cost of capital. Here are some examples.

1. APV provides a natural basis for analysis of the lease vs. buy or lease vs. borrow decision.
2. APV can readily incorporate the impact of dividend policy, if relevant, without making awkward distinctions between the cost of retained earnings vs. the cost of stock issue. Transaction costs in financing can also be accommodated. (The effect of transaction costs on the cost of capital is a relatively complicated function of project life. Under the APV rule, dollar transaction costs are simply subtracted.)
3. Suppose subsidized borrowing is available for certain investments (e.g., for pollution control facilities). How does this affect the investments' value? The impact is clear in the APV framework.

I suggest the reader analyze these cases with and without APV and make his or her own judgment about the concept's usefulness.

V. Concluding Comments

In principle corporate investment and financing decisions should be made simultaneously, since the decisions interact in important ways. This paper presents a framework in which the interactions can be analyzed. Further, the framework has been used to evaluate the most widely accepted weighted average cost of capital formulas, and to derive a more general and flexible capital budgeting rule.

There are other uses for the framework. Specifically, it is possible—given some additional assumptions—to develop a linear programming model that can be of direct assistance to management responsible for overall financial planning. This model is described in another paper written jointly with Professor G. A. Pogue [20].

APPENDIX

Proof That ρ^* Is Independent of Project Profitability When Debt Targets are Specified in Terms of Market Values

Once APV_0 is calculated for a project, then the true cost of capital ρ^* can be calculated via Eq. (7). But there is nothing evident in Eq. (7) that rules out the possibility of ρ^* being a function of the C_t's. It turns out that ρ^* is independent of project profitability only under certain special conditions.

We can restate the cash flows in terms of a scale factor and a pattern over time. That is, $C_t = \gamma_t C$ where $\gamma_1, \gamma_2, \ldots, \gamma_T$ are weight summing to 1. Also, let ρ^*_t be the true cost of capital, under the strong definition of Eq. (7), for the project at some intermediate point $0 < t < T$.

From Eq. (25) and (7), we have for $t = T - 1$

$$APV_{T-1} \equiv C\left[\gamma_{T-1} + \frac{\gamma_T}{1 + \rho^*_{T-1}}\right] = \frac{\gamma_T C/(1 + \rho_o)}{1 - f}. \tag{A.1}$$

Note that this assumes dividend policy is irrelevant (λ^C_t's $= 0$) and that MM are correct—see Eq. (24). The first assumption is necessary to the following proof, but the second is not.

Dividing both sides of Eq. (A.1) by C, we have an expression for ρ^*_{T-1} that is independent of C. Now consider ρ^*_{T-2}, which is defined by

$$APV_{T-2} = C\left[\gamma_{T-2} + \frac{\gamma_T}{1 + \rho^*_{T-1}} + \frac{\gamma_T}{(1 + \rho^*_{T-2})^2}\right] \tag{A.2}$$

But from Eq. (26),

$$APV_{T-2} = \frac{1}{1 - f}\left[A_{T-2} - fC_{T-2} + \frac{f}{1 + r}(APV_{T-1} - C_{T-1})\right]. \tag{A.3}$$

However, all terms within the brackets in Eq. (A.3) are proportional to C. (This is obviously true for A_{T-2}, C_{T-2} and C_{T-1}; we have just shown it to be true for APV_{T-1}). Thus, we can equate Eqs. (A.2) and (A.3), divide through by C, and obtain a definition of ρ^*_{T-2} that is independent of C.

Similarly, ρ^*_{T-3} can be defined in terms of ρ^*_{T-2}, ρ^*_{T-1} and the γ's. By working backwards we eventually find that ρ^* evaluated at $t = 0$ is independent of C. It is also independent of C_o, the initial investment, since C_o is not discounted.

The same result follows if L_t is variable and defined by Eq. (28).

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